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Global Embeddings of Two-dimensional Dilatonic Black Holes

Soon-Tae Hong,* Won Tae Kim,† John J. Oh,‡ and Young-Jai Park§

Department of Physics and Basic Science Research Institute,

Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

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Abstract

We obtain minimal (2+1)- and (2+2)-dimensional global flat embeddings of uncharged and charged dilatonic black holes in (1+1)-dimensions. Moreover, we obtain the Hawking temperatures and the black hole temperatures of these dilatonic black holes. However, even though the minimal flat embedding structures are mathematically meaningful, through these minimal embeddings, the proper entropies are shown to be unattainable, in contrast to the cases of other black holes in (2+1) or much higher dimensions.

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* electronic address: sthong@sogang.ac.kr

† electronic address: wtkim@ccs.sogang.ac.kr

‡ electronic address: john5@string.sogang.ac.kr

§ electronic address: yjpark@sogang.ac.kr

I. INTRODUCTION

There has been tremendous progress in the study of two-dimensional black holes[1] and string theory[2]. It is also well known that in the string theory, a U-duality exists between two-dimensional dilatonic black holes[3, 4, 5, 6] and five-dimensional ones. On the other hand, it is well-known that a thermal Hawking effect on a curved manifold[7] can be viewed at as an Unruh effect[8] in a higher-dimensional flat spacetime. Following the global embedding Minkowski space (GEMS) approach[9], several authors[10, 11, 12, 13] recently have shown that this approach could yield a unified derivation of temperature for various curved manifolds, such as the rotating Banados-Teitelboim-Zanelli (BTZ) manifold[14, 15, 16, 17, 18], Schwarzschild manifold[19] together with its anti-de Sitter (AdS) extension, the Reissner-Nordström (RN) manifold[20] and the RN-AdS manifold[12].

Historically, the higher-dimensional global flat embeddings of black-hole solutions have been subjects of great interest to mathematicians, as well as physicists. In differential geometry, it is well-known that the four-dimensional Schwarzschild metric is not embedded in R^5[21]. Recently, Deser and Levin first obtained (5+1)-dimensional global flat embeddings of the (3+1) Schwarzschild black-hole solution[10]. The (3+1)-dimensional RN-AdS, RN, and Schwarzschild-AdS black holes are also shown to be embedded in (5+2)-dimensional GEMS manifolds[12]. On the other hand, very recently, the brane metric has also been embedded in six dimensions.[22]

Moreover, static, rotating, and charged (2+1)-dimensional BTZ AdS black holes are shown to have (2+2), (2+2), and (3+3) GEMS structures[10, 13], respectively, while the static, rotating, and charged (2+1)-dimensional dS black holes are shown to have (3+1), (3+1) and (3+2) GEMS structures, respectively[13]. Very recently, we have obtained (3+1) and (3+2) GEMS of uncharged and charged (2+1) black strings, respectively[23]. Until now, we have analyzed the GEMS structure of the black hole and black strings in (2+1) and (3+1) dimensions. It is now interesting to study the geometry of (1+1)-dimensional dilatonic black-hole solutions in the GEMS approach to directly yield their minimal flat embeddings.

In this paper, we will analyze the Hawking and Unruh effects of (1+1)-dimensional dilatonic black holes[3, 4, 5] in the framework of the GEMS scheme. In Section II, we will briefly recapitulate two-dimensional dilatonic black holes[3, 4, 5] associated with type IIA strings

to yield asymptotically flat two-dimensional dilatonic black holes. In Section III, we will obtain the minimal GEMS structures of uncharged and charged two-dimensional dilatonic black holes and their corresponding Hawking temperatures. In Section IV, we will discuss the entropies of dilatonic black holes and the embedding constraints in the framework of the GEMS scheme.

II. TYPE IIA STRING THEORY AND TWO-DIMENSIONAL BLACK HOLES

In this section, we briefly recapitulate two-dimensional dilatonic black holes[3, 4, 5] associated with type IIA string theories and their compactification to five dimensions whose metric is the product of a three-sphere and an asymptotically flat two-dimensional geometry. The ten-dimensional type IIA superstring solution consists of a solitonic NS 5-brane wrapping around the compact coordinates, say, x_5, x_i ($i = 6, 7, 8, 9$) and a fundamental string wrapping around x_5, and a gravitational wave propagating along x_5. In the string frame, the 10-metric, dilaton and 2-form field B are given as[24, 25, 26]

$$\begin{aligned} ds^2 &= -(H_1 K)^{-1} f dt^2 + H_1^{-1} K (dx_5 - (K'^{-1} - 1)dt)^2 + H_5 (f^{-1} dr^2 + r^2 d\Omega_3^2) + dx_i dx^i, \\ e^{-2\phi} &= H_1 H_5^{-1}, \quad B_{05} = H_1'^{-1} - 1 + \tanh\alpha, \quad B_{056789} = H_5'^{-1} - 1 + \tanh\beta, \end{aligned} \tag{1}$$

where $r^2 = x_1^2 + \cdots + x_4^2$ and

$$\begin{aligned} H_1 &= 1 + \frac{r_0^2 \sinh^2\alpha}{r^2}, \quad H_5 = 1 + \frac{r_0^2 \sinh^2\beta}{r^2}, \quad K = 1 + \frac{r_0^2 \sinh^2\gamma}{r^2}, \\ H_1'^{-1} &= 1 - \frac{r_0^2 \sinh\alpha\cosh\alpha}{r^2 H_1}, \quad K'^{-1} = 1 - \frac{r_0^2 \sinh\gamma\cosh\gamma}{r^2 K}, \quad f = 1 - \frac{r_0^2}{r^2}. \end{aligned} \tag{2}$$

Here, the B_{05} component of the Neveu-Schwarz 2-form B is the electric field of the fundamental strings, and B_{056789} is the electric field dual to the magnetic field of the 5-brane with components B_{ij}. Exploiting the dimensional reduction in x_5, x_i ($i = 6, 7, 8, 9$) directions in the Einstein frame, one can obtain the five-dimensional black hole metric[24, 25]

$$ds^2 = -(H_1 H_5 K)^{-2/3} f dt^2 + (H_1 H_5 K)^{1/3} (f^{-1} dr^2 + r^2 d\Omega_3^2). \tag{3}$$

On the other hand, performing an $T_5 S T_{6789} S T_5$ transformation with the T-duality $T_{ij\ldots}$ along the $ij\ldots$ directions and the S-duality S of type IIB string[27] and then performing an SL(2,R) coordinate transformation associated with the O(2,2) T-duality group, one can

also obtain the 5-metric

$$\begin{aligned} ds^2 &= -(H_1\bar{H}_5)^{-1}fdt^2 + H_1^{-1}\bar{H}_5(dx_5 - (\bar{H}_5^{-1}-1)dt)^2 \\ &\quad +K(f^{-1}dr^2 + r^2 d\Omega_3^2)(f^{-1}dr^2 + r^2 d\Omega_3^2) \end{aligned} \tag{4}$$

with

$$\bar{H}_5 = \frac{r_0^2}{r^2}. \tag{5}$$

Performing the same set of S and T transformations reversely, one can obtain

$$\begin{aligned} ds^2 &= -(H_1^{-3}\bar{H}_5)^{-1/4}K^{-1}fdt^2 + (H_1^{-3}\bar{H}_5)^{-1/4}K(dx_5 - K'^{-1}-1)dt)^2 \\ &\quad +(H_1\bar{H}_5^3)^{1/4}(f^{-1}dr^2 + r^2 d\Omega_3^2) + (H_1\bar{H}_5^{-1})^{1/4}dx_i dx^i, \end{aligned} \tag{6}$$

$$e^{-2\phi} = \frac{r^2}{r_0^2} + \sinh^2\alpha, \tag{7}$$

which, after exploiting dimensional reduction in the x_5, x_i $(i = 6,7,8,9)$ directions with $\alpha = \gamma$, yield the five-dimensional black hole metric[28]

$$ds^2 = -\left(1 - \frac{r_0^2}{r^2}\right)\left(1 + \frac{r_0^2\sinh^2\alpha}{r^2}\right)^{-2}dt^2 + \left(\frac{r^2}{r_0^2} - 1\right)^{-1}dr^2 + r_0^2 d\Omega_3^2, \tag{8}$$

and the dilaton which is trivially invariant under the dimensional reduction to yield the same result as in Eq. (7). Here, one notes that the metric in Eq. (8) is the product of two completely decoupled parts, namely, a three-sphere and an asymptotically flat a two-dimensional geometry which describes two-dimensional charged dilatonic black hole. Now, introducing a new variable x and the parameters m, q and Q such that

$$e^{Qx} = 2\left(\frac{r^2}{r_0^2} + \sinh^2\alpha\right)(m^2 - q^2)^{1/2}, \quad Q = \frac{2}{r_0}, \tag{9}$$

where m and q are the mass and the charge of the dilatonic black hole, one can obtain the well-known two-dimensional charged dilatonic black hole metric[3, 4]

$$ds^2 = -(1 - 2me^{-Qx} + q^2 e^{-2Qx})dt^2 + (1 - 2me^{-Qx} + q^2 e^{-2Qx})^{-1}dx^2, \tag{10}$$

which will be discussed in the framework of the GEMS scheme in the next section.

III. MINIMAL GEMS STRUCTURES OF TWO-DIMENSIONAL DILATONIC BLACK HOLES

Now, we consider the two-dimensional charged dilatonic black hole having the 2-metric[3, 4]

$$ds^2 = -N^2 dt^2 + N^{-2}dx^2, \tag{11}$$

where the lapse function is given as

$$N^2 = 1 - 2me^{-Qx} + q^2 e^{-2Qx}, \tag{12}$$

from which one can obtain the horizons x_H and x_- in terms of the mass m and the charge q,

$$\begin{aligned} e^{Qx_H} &= m + (m^2 - q^2)^{1/2}, \\ e^{Qx_-} &= m - (m^2 - q^2)^{1/2}. \end{aligned} \tag{13}$$

By using these relations, one can rewrite the lapse function as

$$N^2 = \left(1 - e^{-Q(x-x_H)}\right)\left(1 - e^{-Q(x-x_-)}\right). \tag{14}$$

In order to construct the GEMS structures of the two-dimensional dilatonic black hole, we first consider the uncharged dilatonic black-hole 2-metric

$$ds^2 = -\left(1 - 2me^{-Qx}\right)dt^2 + \left(1 - 2me^{-Qx}\right)^{-1}dx^2. \tag{15}$$

Making an ansatz of two coordinates (z^0, z^1) in Eq. (17) to yield

$$\begin{aligned} -(dz^0)^2 + (dz^1)^2 &= -(1 - e^{-Q(x-x_H)})dt^2 + \frac{e^{-2Q(x-x_H)}}{1 - e^{-Q(x-x_H)}}dx^2 \\ &= ds^2 - (1 + e^{-Q(x-x_H)})dx^2 \equiv ds^2 - (dz^2)^2, \end{aligned} \tag{16}$$

we obtain the (2+1)-dimensional minimal GEMS black hole metric $ds^2 = -(dz^0)^2 + (dz^1)^2 + (dz^2)^2$ given by the coordinate transformations

$$\begin{aligned} z^0 &= k_H^{-1}\left(1 - e^{-Q(x-x_H)}\right)^{1/2}\sinh k_H t, \\ z^1 &= k_H^{-1}\left(1 - e^{-Q(x-x_H)}\right)^{1/2}\cosh k_H t, \\ z^2 &= \frac{2}{Q}\left((1 + e^{-Q(x-x_H)})^{1/2} + \frac{1}{2}\ln\frac{(1 + e^{-Q(x-x_H)})^{1/2} - 1}{(1 + e^{-Q(x-x_H)})^{1/2} + 1}\right), \end{aligned} \tag{17}$$

where $k_H = \frac{Q}{2}$ is the surface gravity and x_H is given by Eq. (13) with $q = 0$. For the trajectories, which follow the Killing vector $\xi = \partial_t$ on the uncharged two-dimensional dilatonic black-hole manifold described by (t, x), one can obtain the constant 2-acceleration,

$$a_2 = \frac{Qe^{-Q(x-x_H)}}{2(1 - e^{-2Q(x-x_H)})^{1/2}}, \tag{18}$$

from the definition of the acceleration in n-dimensional spacetimes, $a_n = \sqrt{a_\alpha a^\alpha}$, where $a_\alpha = \xi^\mu \nabla_\alpha \xi_\mu / |\xi|^2$. Moreover, we can obtain the Hawking temperature and the black-hole temperature :

$$\begin{aligned} T_H &= \frac{a_3}{2\pi} = \frac{Q}{4\pi} \frac{1}{(1-e^{-Q(x-x_H)})^{1/2}}, \\ T &= (-g_{00})^{1/2} T_H = \frac{Q}{4\pi}, \end{aligned} \tag{19}$$

where a_3 is a 3-acceleration in embedded higher-dimensional spacetimes, which is given by $\kappa_H \sqrt{-g^{tt}}$. Here, one notes that the above Hawking temperature is also given by the relation[7]

$$T_H = \frac{1}{2\pi} \frac{k_H}{(-g_{00})^{1/2}}. \tag{20}$$

Next, we consider the charged dilatonic black hole whose 2-metric is given by Eqs. (11) and (12). Similarly to the uncharged case, after some algebra, we arrive at the (2+2) GEMS metric $ds^2 = -(dz^0)^2 + (dz^1)^2 + (dz^2)^2 - (dz^3)^2$ for the charged two-dimensional dilatonic black hole given by the coordinate transformations

$$\begin{aligned} z^0 &= k_H^{-1} \left(1-e^{-Q(x-x_H)}\right)^{1/2} \left(1-e^{-Q(x-x_-)}\right)^{1/2} \sinh k_H t, \\ z^1 &= k_H^{-1} \left(1-e^{-Q(x-x_H)}\right)^{1/2} \left(1-e^{-Q(x-x_-)}\right)^{1/2} \cosh k_H t, \\ z^2 &= \frac{2}{Q} \left(e^{Q(x_H-x_-)/2} \tan^{-1}(e^{-Q(x_H-x_-)} F)^{1/2} + \frac{1}{2} \ln \frac{F^{1/2}-1}{F^{1/2}+1} \right) \\ z^3 &= \frac{2e^{-3Q(x-x_H)/2} e^{-Q(x-x_-)/2}}{Q(e^{-Q(x-x_H)} - e^{-Q(x-x_-)})}, \end{aligned} \tag{21}$$

where the surface gravity k_H and F are given as

$$k_H = \frac{Q}{2}(1-e^{-Q(x_H-x_-)}), \tag{22}$$

$$F = \frac{1+e^{-Q(x-x_H)}}{1-e^{-Q(x-x_-)}}. \tag{23}$$

Here, one can easily check that, in the uncharged limit ($q \to 0$ or $e^{Qx_-} \to 0$), the above coordinate transformations reduce exactly to the previous ones in Eq. (17) for the uncharged dilatonic black hole case.[31]. For the trajectories, which follow the Killing vector $\xi = \partial_t$ on the charged dilatonic black-hole manifold described by (t, x), one can obtain the constant 2-acceleration,

$$a_2 = \frac{Qme^{-Q(x-x_H)}}{(1-e^{-Q(x-x_H)})^{1/2} (1-e^{-Q(x-x_-)})^{1/2} (m + (m^2-q^2)^{1/2})}, \tag{24}$$

and the Hawking temperature and the black-hole temperature,

$$\begin{aligned} T_H &= \frac{a_4}{2\pi} = \frac{Q}{4\pi} \frac{1 - e^{-Q(x_H - x_-)}}{(1 - e^{-Q(x - x_H)})^{1/2}(1 - e^{-Q(x - x_-)})^{1/2}}, \\ T &= (-g_{00})^{1/2} T_H = \frac{Q}{4\pi}(1 - e^{-Q(x_H - x_-)})^{1/2}, \end{aligned} \tag{25}$$

where a_4 is an acceleration in embedded four-dimensional spacetimes.

IV. ENTROPIES OF TWO-DIMENSIONAL DILATONIC BLACK HOLES

In this section, we consider the entropies of the dilatonic black holes in the framework of the GEMS scheme. For the uncharged case, the Rindler horizon condition $(z^1)^2 - (z^0)^2 = 0$ implies $r = r_H$, and the remaining embedding constraints yield $z^1 = f(r)$ where $f(r)$ can be read off from Eq. (17). The area of the Rindler horizon[29] then seems to yield the entropy of the uncharged dilatonic black hole :

$$S = \frac{1}{4G_3} \int \mathrm{d}z^2 \delta(z^2 - f(r)) = \frac{1}{4G_3}, \tag{26}$$

where we have explicitly included the constant of proportionality $1/4G_3$[30]. However, the entropy in Eq. (26) is not equivalent to the previous result in Refs.[4] and[28] since we still have the Newton constant G_3 instead of G_2. Moreover, in defining the entropy (26), we have used an improper constraint condition, $\delta(z^2 - f(r))$, since one needs at least one nontrivial constraint describing a relation among the GEMS coordinates and the constraint in Eq. (26) cannot yield any relation between the coordinates in Eq. (17). In that sense, one cannot obtain the desired proper entropy in the minimal higher-dimensional embeddings of the dilatonic black hole. In order to avoid these difficulties, as a plausible candidate, one could consider other higher-dimensional embeddings such as (3+1)-dimensional GEMS :

$$\begin{aligned} z^0 &= k_H^{-1} \left(1 - e^{-Q(x - x_H)}\right)^{1/2} \sinh k_H t, \\ z^1 &= k_H^{-1} \left(1 - e^{-Q(x - x_H)}\right)^{1/2} \cosh k_H t, \\ z^2 &= x, \\ z^3 &= \frac{2}{Q} e^{-Q(x - x_H)/2}. \end{aligned} \tag{27}$$

Then, using the above nonminimal GEMS and using the relation $G_4 = V_2 G_2 = 2G_2/Q$, where V_2 is a compact volume, $V_2 = 2/Q$, given along z^2 only, one could obtain the desired

entropy :

$$
\begin{aligned}
S &= \frac{1}{4G_4}\int \mathrm{d}z^2\mathrm{d}z^3\delta(z^3-\frac{2}{Q}e^{-Q(z^2-x_H)/2}) \\
&= \frac{1}{4G_4}\int_0^{\frac{2}{Q}}\mathrm{d}z^3=\frac{1}{4G_2},
\end{aligned}
\tag{28}
$$

which is consistent with the previous result in Refs.[4] and[28]. A similar situation happens in the case of the charged two-dimensional dilatonic black hole.

For a charged dilatonic black hole case, for instance, one could consider a (3+2) nonminimal GEMS metric $ds^2=-(dz^0)^2+(dz^1)^2+(dz^2)^2+(dz^3)^2-(dz^4)^2$ given by the coordinate transformations

$$
\begin{aligned}
z^0 &= k_H^{-1}\left(1-e^{-Q(x-x_H)}\right)^{1/2}\left(1-e^{-Q(x-x_-)}\right)^{1/2}\sinh k_Ht, \\
z^1 &= k_H^{-1}\left(1-e^{-Q(x-x_H)}\right)^{1/2}\left(1-e^{-Q(x-x_-)}\right)^{1/2}\cosh k_Ht, \\
z^2 &= x, \\
z_3 &= \frac{2}{Q}(1+e^{Q(x_H-x_-)})^{1/2}\sin^{-1}e^{-Q(x-x_-)/2}, \\
z^4 &= \frac{2e^{-3Q(x-x_H)/2}e^{-Q(x-x_-)/2}}{Q(e^{-Q(x-x_H)}-e^{-Q(x-x_-)})}.
\end{aligned}
\tag{29}
$$

Here, one can also check that, in the uncharged limit, $q\to 0$, the above coordinate transformations reduce exactly to the previous ones in Eq. (27) for the uncharged dilatonic black-hole case. However, with this simplest nonminimal GEMS structure, one cannot obtain a proper entropy consistent with previous results[4, 28]. How many higher dimensions is required to fix the GEMS structure of this charged case to yield the proper entropy is highly nontrivial.

V. CONCLUSIONS

In conclusion, we have investigated the higher-dimensional global flat embeddings of (1+1) uncharged and charged dilatonic black holes. These two-dimensional dilatonic black holes are shown to be minimally embedded in (2+1) and (3+1) dimensions for the uncharged and the charged two-dimensional dilatonic black holes, respectively. In these minimal GEMS, we have obtained the 2-accelerations, the Hawking temperatures, and the black-hole temperatures, which are independent of the dimensionalities of the GEMS structures since they are calculated only in terms of the original metrics (or more practically in terms of the lapse

functions), regardless of the GEMS coordinate transformations. However, even though the minimal GEMS structures are mathematically meaningful, one has difficulties in deriving the GEMS coordinate transformations to yield the proper entropies since the entropies of the dilatonic black holes have nontrivial G_n factors which are associated with the U-duality structure involved in type IIA string theory and depend on the dimensionalities of the GEMS structures. In fact, we have succeeded in obtaining a (3+1) GEMS structure for the uncharged dilatonic black hole that yielded an entropy consistent with previous results without appealing to the somehow complicated U-duality transformations. However, the entropy calculation in the charged case remains unsolved and suggests an open problem. Through further investigation, it will be interesting to study the entropy in the GEMS approach to charged (1+1) dilatonic black holes and to investigate the relations between the GEMS and the U-duality schemes.

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[1] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. **D45**, 1005 (1992); C-J. Ahn, W. T. Kim, Y-J. Park, K. Y. Kim, and Y. Kim, Mod. Phys. Lett. **A7**, 2263 (1992); S. Kojima, N. Sakai, and Y. Tanii, Mod. Phys. Lett. **A10**, 2391 (1995); W. T. Kim, J. Lee, and Y-J. Park, Phys. Lett. **B347**, 217 (1995); T. Futamase, M. Hotta, Y. Itoh, Phys. Rev. **D57**, 1129 (1998); I. Oda, Phys. Rev. **D57**, 2415 (1998); W. Kummer, Annalen Phys. **8**, 801 (1999); S-W. Kim and H. Lee, J. Korean Phys. Soc. **35**, S666 (1999); D. Youm, Phys. Rev. **D61**, 044013 (2000); D. Grumiller, D. Hofmann, and W. Kummer, Annals Phys. **290**, 69 (2001); S. Nojiri and S. D. Odintsov, Int. J. Mod. Phys. **A16**, 1015 (2001).

[2] G. Mandal, A. M. Sengupta, and S. R. Wadia, Mod. Phys. Lett. **A6**, 1685 (1991); S. Elitzur, A. Forge, and E. Rabinovici, Nucl. Phys. **B359**, 581 (1991); E. Witten, Phys. Rev. **D44**, 314 (1991).

[3] M. D. McGuigan, C. R. Nappi, and S. A. Yost, Nucl. Phys. **B375**, 421 (1992).

[4] C. R. Nappi and A. Pasquinucci, Mod. Phys. Lett. **A7**, 3337 (1992).

[5] G. W. Gibbons and M. J. Perry, Int. J. Mod. Phys. **D1**, 335 (1992).

[6] S. Hyun, J. Korean Phys. Soc. **33**, S532 (1998), `hep/th-9704005`.

[7] S. W. Hawking, Commun. Math. Phys. **42**, 199 (1975); J. D. Bekenstein, Phys. Rev. **D7**, 2333 (1973); R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics* (The University of Chicago Press, Chicago, 1994); J. D. Brown, C. Creighton, and R. B. Mann, Phys. Rev. **D50**, 6394 (1994).

[8] W. G. Unruh, Phys. Rev. **D14**, 870 (1976); P. C. W. Davies, J. Phys. **A8**, 609 (1975).

[9] E. Kasner, Am. J. Math. **43**, 130 (1921); C. Fronsdal, Phys. Rev. **116**, 778 (1959); H. F. Goenner, *General Relativity and Gravitation* (Plenum, New York, 1980) Ed. A. Held; J. Rosen, Rev. Mod. Phys. **37**, 204 (1965); H. Narnhofer, I. Peter, and W. Thirring, Int. J. Mod. Phys. **B10**, 1507 (1996).

[10] S. Deser and O. Levin, Class. Quantum Grav. **14**, L163 (1997); Class. Quantum Grav. **15**, L85 (1998); Phys. Rev. **D59**, 0640004 (1999).

[11] M. Beciu and H. Culetu, Mod. Phys. Lett. **A14**, 1 (1999); P. F. Gonzalez-Diaz, Phys. Rev. **D61**, 024019 (1999); L. Andrianopoli, M. Derix, G. W. Gibbons, C. Herdeiro, A. Santambrogio, and A. V. Proeyen, Class. Quant. Grav. **17**, 1875 (2000).

[12] Y-W. Kim, Y-J. Park, and K. S. Soh, Phys. Rev. **D62**, 104020 (2000).

[13] S-T. Hong, Y-W. Kim, and Y-J. Park, Phys. Rev. **D62**, 024024 (2000); S-T. Hong, W. T. Kim, Y-W. Kim, and Y-J. Park, Phys. Rev. **D62**, 064021 (2000).

[14] M. Banados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992); M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. **D48**, 1506 (1993).

[15] S. Carlip, Class. Quant. Grav. **12**, 2853 (1995).

[16] S. W. Kim, W. T. Kim, Y-J. Park, and H. Shin, Phys. Lett. **B392**, 311 (1997).

[17] D. Cangemi, M. Leblanc, and R. B. Mann, Phys. Rev. **D48**, 3606 (1993).

[18] J. Ho, W. T. Kim, and Y-J. Park, J. Korean Phys. Soc. **33**, S541 (1998); H-C. Kim, Y. Kim, and P. Oh, J. Korean Phys. Soc. **35**, S655 (1999); T. Lee, J. Korean Phys. Soc. **35**, S670 (1999).

[19] K. Schwarzschild, Sitzber. Deut. Akad. Wiss. Berlin, KI. Math.-Phys. Tech., pp. 189-196 (1916).

[20] H. Reissner, Ann. Physik **50**, 106 (1916); G. Nordstrom, Proc. Kon. Ned. Akda. Wet **20**, 1238 (1918); C. S. Peca and J. P. S. Lemos, Phys. Rev. **D59**, 124007 (1999); P. Mitra, Phys. Lett. **B459**, 119 (1999); S. W. Hawking and H.S. Reall, Phys. Rev. **D61**, 024014 (1999); B. Wang, E. Abdalla, and R-K. Su, Phys. Rev. **D62**, 047501 (2000).

[21] M. Spivak, *Differential Geometry* (Publish or Perish, Berkeley, 1975) Vol 5, Chapter 11.

[22] M. Gogberashvili, gr-qc/0202061.

[23] S-T. Hong, W. T. Kim, J. J. Oh, and Y-J. Park, Phys. Rev. **D63**, 127502 (2001).

[24] G. T. Horowitz, J. M. Maldacena, and A. Strominger, Phys. Lett. **B383**, 151 (1996).

[25] A. A. Tseytlin, Mod. Phys. Lett. **A11**, 689 (1996).

[26] J. M. Maldacena, *Black holes in string theory*, Ph.D Thesis (Princeton University), hep-th/9607235.

[27] E. Bergshoeff, C. Hull and T. Ortin, Nucl. Phys. **B451**, 547 (1995).

[28] E. Teo, Phys. Lett. **B430**, 57 (1998).

[29] G. W. Gibbons and S. W. Hawking, Phys. Rev. **D15**, 2738 (1977); R. Laflamme, Phys. Lett. **B196**, 449 (1987).

[30] S. W. Hawking, Nature **248**, 30 (1974).

[31] For the case of z^2, one needs to exploit L'Hospital's rule to explicitly see the uncharged limit.